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Current states in a metal plate

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Abstract. The effect of current rectification in a metal plate, placed in a parallel magnetic field h_0 and irradiated by radio waves of sufficiently large amplitude $\mathcal{H}$, is investigated theoretically. On the basis of Pippard's ineffectiveness concept, the equation for the induced magnetic field h inside the sample is obtained. The dependence $h(h_0)$ exhibits hysteresis behaviour for amplitudes $\mathcal{H}$ larger than a certain critical value $\mathcal{H}_{cr}$. We show that on diminishing the plate thickness d, the induced field h decreases, whereas the critical amplitude $\mathcal{H}_{cr}$ augments. On increasing the amplitude $\mathcal{H}$ of the radio wave, the hysteresis loops $h(h_0)$ for a thin plate converge to a limit curve determined by sample parameters: the skin depth δ and the thickness d.

1. Introduction

For many years investigations of non-linear electromagnetic properties of solid-state plasmas have been of great interest. As a result of these investigations, it was established that many well known non-linear electrodynamic effects, associated with an electric field and existing in semiconductors, are not observed in normal metals. This is understood in terms of the fact that the electric field is always weak in metals, due to their high electrical conductivity. However, the same cause that inhibits the manifestation of common non-linear properties is responsible for the appearence of a new kind of non-linear effects in metals. Indeed, in pure metal samples, especially at low temperatures, the magnetic component of the electromagnetic field comes to the fore because of the high conductivity of pure metals. Thus, the Lorentz force determined by the magnetic field of a wave influences the shape of electron trajectories, and consequently the sample conductivity. This mechanism of non-linearity is called magnetodynamic. The first experimental reports on magnetodynamic non-linearity were presented in the papers [1] and [2].

The magnetodynamic mechanism of non-linearity is very effective under conditions of the typical (for metals) anomalous skin effect, when the skin depth δ_a is much smaller than the electron mean free path l and the radius of curvature R of the electron trajectories in the magnetic field of the wave, i.e.,

$$\delta_a \ll l \qquad \delta_a \ll R \qquad \delta_a = (lc^2/3\pi^2\sigma_0\omega)^{1/3} \qquad R = cp_F/e\mathcal{H}. \tag{1}$$

Here $\mathcal{H}$ and ω are the amplitude and the frequency, respectively, of the radio wave, σ_0 is the static conductivity of the metal, p_F is the Fermi momentum, e is the absolute value of the electron charge, and c is the speed of light in vacuum.

The effectiveness of the non-linearity mechanism is characterized by the ratio of the mean free path l to the electron-trajectory length in the skin layer, $(8R\delta_a)^{1/2}$ (see review [3]):

$$b = l/(8R\delta_a)^{1/2} = (\mathcal{H}/\tilde{h})^{1/2} \qquad \tilde{h} = 8cp_F\delta_a/el^2. \tag{2}$$

The field $\tilde{h}$ corresponds to the value of the amplitude $\mathcal{H}$ when the bending of electron trajectories becomes significant and non-linear effects begin to be manifested. For pure metal samples, over the radio-wave frequency band ($\delta_a \simeq 10^{-4} - 10^{-3}$ cm) and at helium temperatures ($l \simeq 0.1$ cm), the field $\tilde{h}$ has a small value, 0.5–5 Oe. In the experiments, the value of the electromagnetic-wave amplitude $\mathcal{H}$ reaches a few tens or hundreds of Oersteds. For this reason, not only weak non-linearity ($b \ll 1$), but also strong non-linearity ($b \gg 1$) is experimentally realizable.

Because of the magnetodynamic non-linearity, a great many non-linear phenomena can be observed in metals (see, for instance, [3–6] and references therein). Among these phenomena, the excitation of current states is one of the most notable. This is a singular hysteresis effect of rectification of a high-frequency current and of the appearence of the intrinsic magnetic moment of the sample. This effect was discovered in bismuth [7, 8] and later observed in other metals [9–11]. In the experiments, pure samples were irradiated by radio waves ($\omega \simeq 10$–10^6 s^{-1}) of sufficiently large amplitude ($\mathcal{H} \simeq 1$–10^3 Oe). The waves generated a closed constant current in the metal, which induced a constant magnetic field h and, hence, a magnetic moment $M = h/4\pi$ of the sample (figure 1). In order to excite the current states, a weak constant magnetic field h_0 should be applied parrallel to the magnetic field of the radio wave $\boldsymbol{H}$. Nevertheless, the magnetic moment as a function of h_0 exhibits a hysteresis behaviour and in the case of developed non-linearity ($b \geqslant 1$) it may remain finite even when h_0 vanishes. The effect of excitation of current states has an excitation threshold: hysteresis loops $h(h_0)$ are formed only when the amplitude $\mathcal{H}$ exceeds the critical value $\mathcal{H}_{cr}$, which is of the order of a few Oersteds according to the experiments.

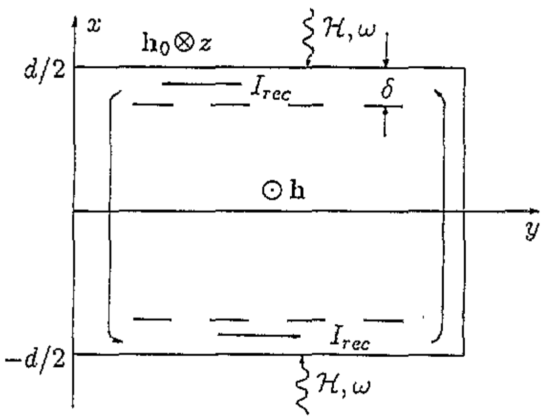

Figure 1. Schematic representation of the experimental geometry. I_{rec} is the rectified current, h_0 and h are the external and the induced magnetic fields respectively, $\mathcal{H}$ and ω are the amplitude and the frequency, respectively, of the radio wave, and δ is the characteristic skin depth.

The mechanism of current rectification was suggested in [12]. Here it was noted that the non-uniform magnetic field in the metal, which determines the dynamics of electrons, is the sum of the magnetic field $\boldsymbol{H}(x, t)$ of the wave and the external field $\boldsymbol{h}_0$ ($\boldsymbol{h}_0 \parallel \boldsymbol{H}(x, t)$; the x axis coincides with the direction of radio-wave propagation). The character of the electron movement depends completely on the existence of a plane $x = x_0(t)$ in the sample, where the total magnetic field $\boldsymbol{H} + \boldsymbol{h}_0$ becomes zero. In the time intervals when this plane is present, the spatial distribution of the total magnetic field alternates in sign. As result, a group of trapped electrons, which is specific only for the non-linear regime, appears. The trajectories of these electrons wind around the plane $x = x_0(t)$ precisely in the vicinity of the sample boundaries. Consequently, during the whole mean free time, the trapped electrons interact with the electromagnetic field in the metal and determine the sample conductivity. When the plane $x = x_0$ is absent, the trapped electrons disappear and, therefore, the metal conductivity is significantly diminished. The changes of the electron conductivity occurring in each wave period lead to the effect of current rectification and to the induction of a constant inhomogeneous magnetic field $h(x)$. The induced field $h(x)$ varies over distances of order δ_a from zero at the metal boundaries ($h(\pm d/2) = 0$, see figure 1) to its maximum value at the middle of the current loop.

Until now, the theory of current states has been constructed only for the case of samples having infinite thickness [3]. Within the framework of this theory the threshold character of current state generation was established and the threshold amplitude of the exciting radio wave was calculated. In addition, the dynamics of development of current state hysteresis loops was investigated. In particular, it was found that, with increasing radio-wave amplitude $\mathcal{H}$, the dependence of the induced magnetic field h on the external magnetic field h_0 tends to a universal function, defined by $\mathcal{H}$ and independent of other task parameters. Nevertheless, in experimental conditions the sample thickness d is always finite. Moreover, in some experimental works d only just exceeds the skin depth δ (see, for instance, [11] and [13]). In these and other works the measured values of the threshold amplitudes $\mathcal{H}_{cr}$ turned out to be notably larger than theoretically calculated ones. In addition, there are no experiments which would indicate the coincidence of the hysteresis loop $h(h_0)$ with the predicted universal curve. These facts can be explained by the theory, taking into account the finiteness of the sample thickness. The present work is devoted to formulating such a theory.

The next section contains the foundation of the theory based on the ineffectiveness concept in the non-linear electrodynamics of metals. In section 3 we calculate the induced magnetic field h for a finite plate. Here the variations of the dependence $h(h_0)$, and of the threshold amplitude $\mathcal{H}_{cr}$, with diminishing thickness d are analysed.

2. Formulation of the problem

Let us analyse the bilateral excitation of a metal plate, having thickness d by a radio wave with amplitude $\mathcal{H}$ and frequency ω. The x axis is oriented perpendicular to the plate boundaries and the plane $x = 0$ is placed in the middle of the plate. The external constant and homogenous magnetic field h_0 is applied along the z axis (figure 1). The magnetic field $\boldsymbol{H}$ of the radio wave in the metal is collinear with h_0.

It is well known that electromagnetic waves propagate themselves in a metal practically perpendicular to its surface, independent of the incidence angle. According to the chosen geometry, Maxwell's equations can be expressed as follows:

$$-\partial H(x,t)/\partial x = (4\pi/c)j(x,t) \qquad \partial E(x,t)/\partial x = -(1/c)\partial H(x,t)/\partial t. \tag{3}$$

The boundary conditions for these equations are

$$H(\pm d/2, t) = \mathcal{H}\cos\omega t. \tag{4}$$

Hence, the magnetic field $H(x,t)$ should be an even function with respect to x while the electric field $E(x,t)$ is odd function.

Solving the system of equations (3) we can calculate the induced magnetic field $h(x)$ which is equal to the value of $H(x,t)$ averaged over the wave period $2\pi/\omega$:

$$h(x) = \langle H(x,t)\rangle \equiv \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \mathrm{d}t\, H(x,t). \tag{5}$$

The mean value (over the plate thickness) of the induced magnetic field,

$$h \equiv \frac{1}{d}\int_{-d/2}^{d/2} \mathrm{d}x\, h(x) \tag{6}$$

is the quantity measured in the experiments. Therefore, we shall calculate and analyse this quantity.

In order to solve analytically the system of equations (3) we shall find the non-linear relation between the current density $j(x,t)$ and the electromagnetic field in the metal plate with the aid of the Pippard's ineffectiveness concept. This method was extremely useful for the case $d \to \infty$ [3, 14]. The ineffectiveness concept is based on the fact that the anomalous skin effect involves only a small fraction of electrons, which are called effective. Their conductivity σ_{eff} can be estimated by using the simple formula

$$\sigma_{\mathrm{eff}} = N_{\mathrm{eff}} e^2 \tau/m \tag{7}$$

where N_{eff} is the number of effective electrons arriving at the skin layer, τ is the time of their interaction with the electromagnetic field in this layer, and m symbolizes the electron mass. As a consequence of the spatial dispersion, the conductivity σ_{eff} turns out to be dependent on the skin depth δ and, thus, Maxwell's equations (3) can be transformed into an equation for δ. However, in the non-linear regime the conductivity of the effective electrons depends not only on δ but also on the time t. For this reason, the Pippard ineffectiveness concept has to be generalized for the non-linear case. Below, we shall deal with this generalized model.

Firstly, we must separate space and time variables in (3). The expansion into a Fourier harmonic series in ωt does not help since all the harmonics in the sample, under developed non-linearity, have practically the same order of magnitude and strongly interact with one another. It is much more convenient to write the electric and magnetic fields in the metal as

$$E(x,t) = \sum_{n=-\infty}^{\infty} E_n(\phi)\exp[-\mathrm{i}n\xi(\phi)]\frac{\sinh(x/\delta^{(n)})}{\cosh(d/2\delta^{(n)})}$$
$$H(x,t) = \sum_{n=-\infty}^{\infty} H_n\exp[-\mathrm{i}n\xi(\phi)]\frac{\cosh(x/\delta^{(n)})}{\cosh(d/2\delta^{(n)})} \qquad \phi = \omega t. \tag{8}$$

The quantities $E_n(\phi)$, $\delta^{(n)}$ and $\xi(\phi)$ will be henceforward determined from Maxwell's equations (3), and the coefficients H_n, which are independent of time, from the boundary conditions (4). In view of the periodicity in time of the fields (8), the function $\xi(\phi)$ must satisfy the condition

$$\xi(\phi + 2\pi) = \xi(\phi) + 2\pi. \tag{9}$$

Furthermore, $\xi(\phi)$ ought to be continous, monotonic, and single valued so that the set of functions $\exp[-\mathrm{i}n\xi(\phi)]$ in (8) will be complete.

As in the representation (8), the current density of conduction electrons can be expressed in the form

$$j(x,t) = \sum_{n=-\infty}^{\infty} \sigma^{(n)}(\phi) E_n(\phi) \exp[-in\xi(\phi)] \frac{\sinh(x/\delta^{(n)})}{\cosh(d/2\delta^{(n)})}. \tag{10}$$

The conductivity $\sigma^{(n)}(\phi)$ is determined by distinct electron groups depending on the phase of the exciting radio wave. If the wave amplitude is larger than the external magnetic field,

$$h_0 < \mathcal{H} \tag{11}$$

there exists a time interval in each wave period $2\pi/\omega$ when the following inequality is satisfied

$$[\mathcal{H}\cos(\omega t) + h_0](h + h_0) < 0. \tag{12}$$

During this time the spatial distribution of the total magnetic field alternates in sign and the conductivity $\sigma^{(n)}(\phi)$ represents the sum

$$\sigma^{(n)} = \sigma_{\rm trap}^{(n)} + \sigma_{\rm s}^{(n)}. \tag{13}$$

Here, $\sigma_{\rm trap}^{(n)}$ is the conductivity of trapped electrons mentioned in section 1 (see also figure 2(*a*)). Let us estimate this value using equation (7). The time that the trapped electron stays in the skin layer in one period of its twisted motion is of the order of $\tau_0 = (R\delta^{(n)})^{1/2}/v_{\rm F}$ ($v_{\rm F}$ is the electron Fermi velocity). Such electrons have a momentum component normal to the metal surface $|p_x| \simeq p_{\rm F}(\delta^{(n)}/R)^{1/2}$, and $N_{\rm eff}$ turns out to be equal to $N(\delta^{(n)}/R)^{1/2}$ (N is the electron density). We must now take into account the repeated return of the trapped electrons to the skin layer. In the quasistatic situation,

$$\omega \ll \nu \tag{14}$$

it is not necessary to allow the change of phase of the electromagnetic wave and the probability of the next return in sequence is equal to $\exp(-2\nu T_{\rm trap})$. Therefore, the estimation (7) for trapped electrons gives

$$\sigma_{\rm trap}^{(n)} = \frac{Ne^2}{m}\frac{(R\delta^{(n)})^{1/2}}{v_{\rm F}}\left(\frac{\delta^{(n)}}{R}\right)^{1/2}\left(1 + 2\sum_{n=1}^{\infty}\exp(-2n\nu T_{\rm trap})\right) = \sigma_0 \frac{\delta^{(n)}}{l}\coth(\nu T_{\rm trap}). \tag{15}$$

In this formula ν is the frequency of bulk collisions, and $2T_{\rm trap}$ is the characteristic value of the period of the trapped electrons:

$$2T_{\rm trap} = (b\nu)^{-1}\mathcal{H}/|h_0 + h|. \tag{16}$$

The quantity b is the non-linearity parameter defined in section 1 (equation (2)). The presence of the factor of two in front of the summation sign in equation (15) is due to the fact that the time of the 'first' sojourn in the skin layer turns out to be half as large as that of the succeeding ones. Note that the spatial dispersion in the conductivity (15) is taken into account by the factor $\delta^{(n)}/l$ which reflects the fact that not all electrons, but only those present in the skin layer $\delta^{(n)}$, interact with the electromagnetic field. The quantity

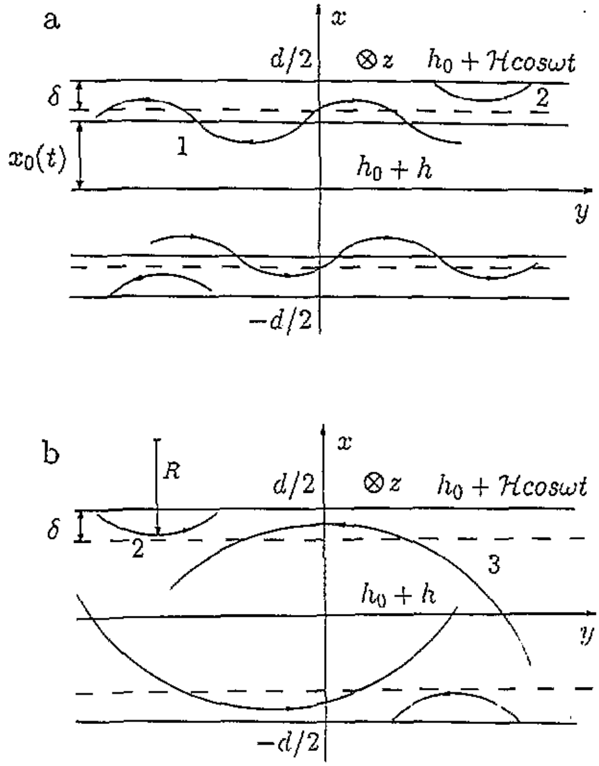

Figure 2. Trajectories of trapped (1), skipping (2), and Larmor (3) electrons in a sign-alternating (*a*) and constant (*b*) total magnetic field.

$\coth(\nu T_{\rm trap})$ in the expression for the conductivity of trapped electrons is related to the probability of their multiple returns to the skin layer [3]. Formula (15) is valid when

$$3\delta < d. \tag{17}$$

In this case the trapped electrons from one skin layer cannot pass to another skin layer (see figure 2(*a*)).

The second term in the right-hand side of equation (13) denotes the conductivity of so-called skipping electrons. Such electrons move along the sample boundaries, remaining inside the skin layer and undergoing collisions with the metal surface (see figure 2). Their number $N_{\rm eff}$ coincides with the number of trapped electrons. For diffuse reflection from the boundaries of the plate, the time τ of skipping-electron interaction with the electromagnetic field in the skin layer is τ_0. So, the conductivity $\sigma_{\rm s}^{(n)}$ of skipping electrons is given by the following formula:

$$\sigma_{\rm s}^{(n)} = \sigma_0 \delta^{(n)}/l. \tag{18}$$

During another part of the wave period $2\pi/\omega$, when the inequality (12) changes for the opposite one, the group of trapped electrons disappears. In this situation a new group of so-called Larmor electrons together with the skipping electrons contributes to the metal conductivity:

$$\sigma^{(n)} = \sigma_{\rm L}^{(n)} + \sigma_{\rm s}^{(n)}. \tag{19}$$

Unlike the trapped electrons, the Larmor ones do not return to the skin layer because the spatial distribution of the total magnetic field has a constant sign. After leaving the skin layer, the Larmor electrons continue moving into the bulk of the sample along the Larmor arc in the constant magnetic field $h + h_0$ (see figure 2(*b*)). If the characteristic radius R of the Larmor arc is larger than the mean free path l,

$$R \geqslant l \tag{20}$$

the conductivity of Larmor electrons is of the same order as that of the skipping ones (18).

As a result, the total time-dependent conductivity of all electron groups can be written in the form

$$\begin{aligned} &\sigma^{(n)}(\phi) = 2\sigma_0(\delta^{(n)}/l)S(\phi) \qquad S(\phi) = 1 + \alpha\Theta[-(\mathcal{H}\cos\phi + h_0)/(h+h_0)] \\ &\alpha = \{\exp[(b|\tilde{\kappa}|)^{-1}] - 1\}^{-1} \qquad \tilde{\kappa} = (h+h_0)/\mathcal{H}. \end{aligned} \tag{21}$$

Here $\Theta(x)$ is the Heaviside function and the quantity α represents the relative change in the conductivity at the moments when the group of trapped electrons appears and disappears.

3. Equation for the induced magnetic field

Let us substitute the expressions for the fields $E(x,t), H(x,t)$ (8), the current density $j(x,t)$ (10), and the conductivity $\sigma^{(n)}(\phi)$ (21) into Maxwell's equations (3). We obtain the function

$$\begin{aligned} &\xi(\phi) = \frac{1}{\mu}\int_0^{\phi}\frac{\mathrm{d}\phi'}{S(\phi')} = \frac{\alpha+2}{\alpha+1}\frac{\phi}{2\mu} + \frac{\alpha\,\mathrm{sign}\,\tilde{\kappa}}{2\mu(\alpha+1)}\begin{cases}\phi & -\beta<\phi<\beta \\ 2\beta-\phi & \beta<\phi<2\pi-\beta\end{cases} \\ &\beta = \cos^{-1}(-h_0/\mathcal{H}) \qquad \mu = \frac{1}{2\pi}\oint\frac{\mathrm{d}\phi}{S(\phi)} = 1 - \frac{\cos^{-1}[\mathrm{sign}(\tilde{\kappa})h_0/\mathcal{H}]}{\pi(1+\alpha^{-1})} \end{aligned} \tag{22}$$

and the quantity $\delta^{(n)}$:

$$\delta^{(n)} = \delta|n|^{-1/3}\exp\left(\frac{\mathrm{i}\pi n}{6|n|}\right) \qquad \delta = \delta_a\mu^{1/3}. \tag{23}$$

Here and below the value δ_a differs from equation (1) by the factor $(3\pi/8)^{1/3}$. This difference is connected with the fact that here we construct a theory based on the ineffectiveness concept. It is well known that such theories give correct results to within real-valued positive factors of the order of unity. According to equations (8), (10) and (21)–(23), the amplitudes of the electric and magnetic fields are related by the formula

$$E_n(\phi) = [\mathrm{i}n\omega\delta^{(n)}/c\mu S(\phi)]H_n. \tag{24}$$

The coefficients H_n must be found from the boundary condition (4). Thus,

$$\sum_{n=-\infty}^{\infty} H_n\exp[-\mathrm{i}n\xi(\phi)] = \mathcal{H}\cos\phi. \tag{25}$$

The expression (25) corresponds to an expansion of the function $\mathcal{H}\cos\phi$ into a series in the complete set of functions $\exp[-\mathrm{i}n\xi(\phi)]$. These functions are orthogonal in the interval

$0 \leqslant \phi \leqslant 2\pi$ with weight $\partial\xi(\phi)/\partial\phi = 1/\mu S(\phi)$. For this reason, the series coefficients H_n are defined as follows:

$$H_n = \frac{\mathcal{H}}{2\pi\mu}\int_0^{2\pi}\frac{d\phi}{S(\phi)}\cos\phi\exp[in\xi(\phi)]. \tag{26}$$

The formula (8) for the magnetic field $H(x,t)$ together with equations (5) and (6) leads to an equation for the mean (over the plate thickness) induced magnetic field h, which can be written in the form

$$h = \frac{2}{d}\sum_{n=-\infty}^{\infty} H_n\delta^{(n)}\tanh(d/2\delta^{(n)})\langle\exp[-in\xi(\phi)]\rangle. \tag{27}$$

Taking into account the fact that $\delta^{(0)} = \infty$, it is convenient to separate the term with $n = 0$ in (27), this being equal to H_0. Due to the fact that this term coincides with the value of h calculated for the case $d \to \infty$ [3], we shall symbolize it as $h_\infty \equiv H_0$. Then,

$$h = h_\infty + \frac{4\delta}{d}\mathrm{Re}\sum_{n=1}^{\infty}\tanh\left(\frac{d}{2\delta^{(n)}}\right)\frac{H_n}{n^{1/3}}\langle\exp[-in\xi(\phi)+i\pi/6]\rangle. \tag{28}$$

The second term in the right-hand side of equation (28) takes into account the finiteness of the sample thickness. We shall consider this term as a linear function of δ/d because the factors $\tanh(d/2\delta^{(n)})$ are not in practice different from unity under condition (17).

After calculating the integrals with respect to ϕ in (28), the equation for the induced magnetic field can be rewritten as

$$\kappa = (1-a^2)^{1/2}\,\mathrm{sign}\,\tilde{\kappa}/\pi\mu(1+\alpha^{-1}) - (\delta_a/d)A \tag{29}$$

where we have introduced the notation

$$\kappa = h/\mathcal{H} \qquad a = h_0/\mathcal{H}$$

$$A = \frac{3^{1/2}}{\pi^2}\alpha^2\mu^{1/3}\sum_{n=1}^{\infty}\left[\sin\left(\frac{n\beta}{\mu S_1}\right)\Big/ n^{4/3}\right] \times\sum_{r=\pm n}\left\{\sin\left[\beta\left(1+\frac{r}{\mu S_1}\right)\right]\Big/\left(S_1+\frac{r}{\mu}\right)\left(S_2+\frac{r}{\mu}\right)\right\}. \tag{30}$$

Here the quantities

$$S_1 = 1+\alpha(1-\mathrm{sign}\,\tilde{\kappa})/2 \qquad \text{and} \qquad S_2 = 1+\alpha(1+\mathrm{sign}\,\tilde{\kappa})/2 \tag{31}$$

coincide with the values of $S(\phi)(21)$ in the intervals $-\beta < \phi < \beta$ and $\beta < \phi < 2\pi - \beta$ respectively.

The equation of the induced field κ (29) was numerically solved for different values of the non-linearity parameter b. The results of this calculation for a plate of finite thickness and for the case $\delta/d = 0$ are shown in figure 3. In this figure we can see that the mean value of the induced magnetic field h for a plate of finite size is smaller than h_∞. Figure 3 also shows that if the parameter b does not exceed a certain critical value b_{cr} then the dependence $\kappa(a)$ (or $h(h_0)$) is single valued. For $b = b_{cr}$, points with vertical tangents appear on the

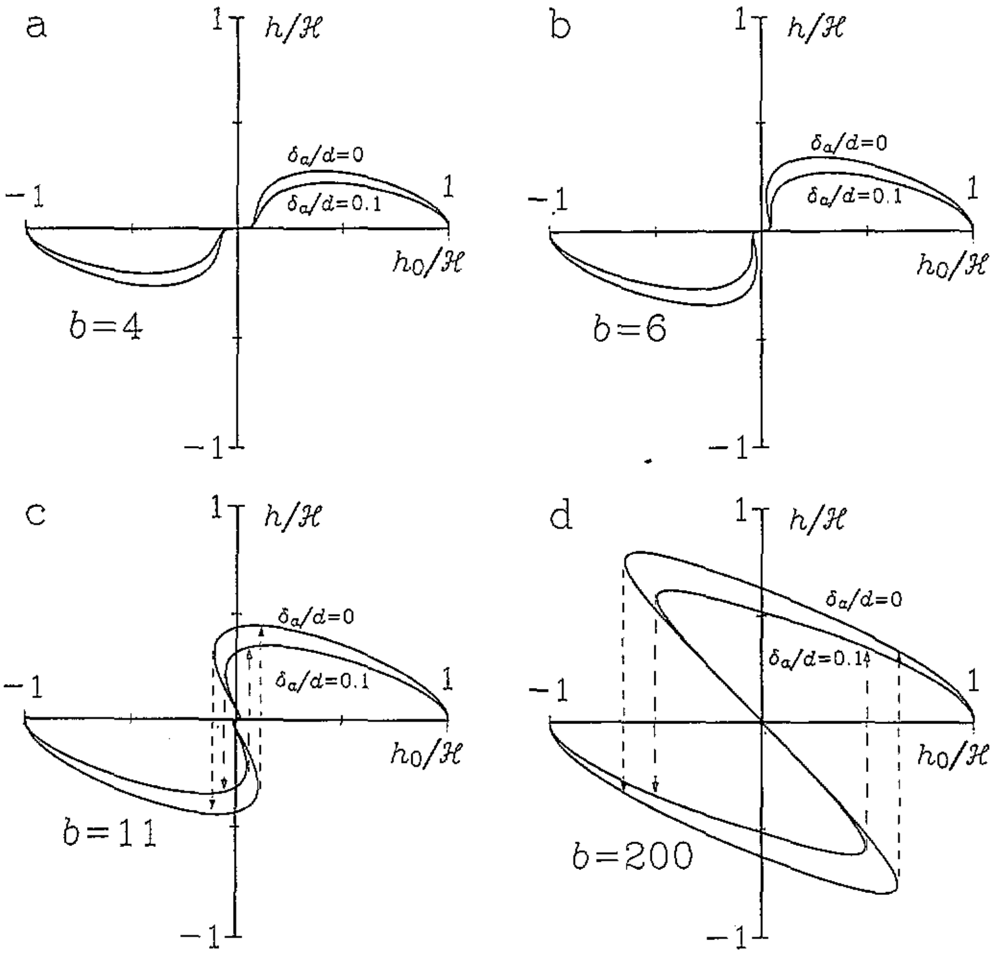

Figure 3. Dynamics (a)–(d) of evolution of current-state hysteresis ($h(h_0)$) with increasing non-linearity parameter b in the case of diffuse reflection of electrons from the surface for $\delta_a/d = 0$ and for $\delta_a/d = 0.1$.

curves $\kappa(a)$. The instability of the current state on the interval between the points, where $\partial\kappa/\partial a = \infty$, originates jumps in $\kappa(a)$ at these singular points. Thus, for $b > b_{cr}$, the $\kappa(a)$ are not single valued and hysteresis loops of the induced magnetic field are formed.

Let us analyse the dependence of the critical value b_{cr} on the thickness d. In figure 3(b) the functions $\kappa(a)$ with $b = 6$ are shown at two different values of δ_a/d. It is clearly seen that the dependence $\kappa(a)$ is multiple valued for $\delta_a/d = 0$, i.e. hysteresis takes place. However, $\kappa(a)$ is single valued for $\delta_a/d = 0.1$, and hysteresis is absent. This means that the finiteness of the plate thickness leads to the shift of the current-state excitation threshold to larger values of the non-linearity parameter b (of the incident wave amplitude $\mathcal{H}$). This conclusion is confirmed by the direct numerical calculation of the quantity b_{cr} as a function of δ_a/d using equation (29). The corresponding plot is presented in figure 4. It is noteworthy that equation (29) for the induced magnetic field h allows us to study the dependence $b_{cr}(\delta_a/d)$ by an analytical method too. When $b = b_{cr}$, the $\kappa(a)$ curve has jumps at two points $(\pm a_{cr}, \pm\kappa_{cr})$ where the equalities $\partial a/\partial\kappa = 0$, $\partial^2 a/\partial\kappa^2 = 0$ are satisfied. These two relations together with equation (29) form a system which allows us

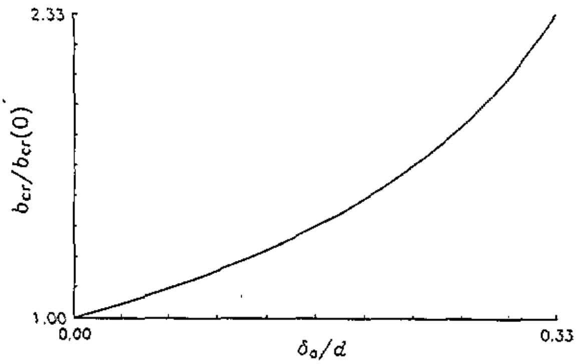

Figure 4. Dependence of the critical value of the non-linearity parameter on the ratio δ_a/d.

to determine the 'critical' values a_{cr}, κ_{cr} and b_{cr}:

$$\begin{aligned}
&\pi[(1+\alpha^{-1})-\tfrac{1}{2}] = (1/\kappa)(1-3^{1/2}\delta_a/d)\\
&\pi(\kappa/\tilde{\kappa})^2(1+\alpha^{-1}) = b(1-3^{1/2}\delta_a/d)\\
&(\kappa/\tilde{\kappa})(1+1/2b\tilde{\kappa}) = 1.
\end{aligned} \tag{32}$$

Here we have taken into account the fact that a_{cr} and κ_{cr} are very small ($a_{cr} \ll 1, \kappa_{cr} \ll 1$), and A (30) with accuracy to

$$\tfrac{1}{6}(1+\alpha^{-1})^{-1} \simeq \tfrac{1}{6}\exp(-2) \ll 1 \tag{33}$$

is given by

$$A = (3^{1/2}/\pi)\alpha\mu^{4/3} \simeq 3^{1/2}/[\pi(1+\alpha^{-1})-\pi/2]. \tag{34}$$

The solution of the system (32) may be expressed in the following form

$$\begin{aligned}
&a_{cr}(\delta_a/d) = a_{cr}(0)(1-3^{1/2}\delta_a/d)\\
&\kappa_{cr}(\delta_a/d) = \kappa_{cr}(0)(1-3^{1/2}\delta_a/d)\\
&b_{cr}(\delta_a/d) = b_{cr}(0)/(1-3^{1/2}\delta_a/d).
\end{aligned} \tag{35}$$

Here, the quantities $a_{cr}(0)$, $\kappa_{cr}(0)$, and $b_{cr}(0)$ determine the hysteresis threshold of the current states in an infinite metal plate and satisfy the system (32) with $\delta_a/d = 0$. These quantities are approximately equal to

$$a_{cr}(0) = 0.055 \qquad \kappa_{cr}(0) = 0.066 \qquad b_{cr}(0) = 5.00. \tag{36}$$

Thus, the consideration of the finite size of the sample leads to the augmenting of the critical value b_{cr} and to the displacement of the critical points ($\pm a_{cr}, \pm\kappa_{cr}$) to the coordinate origin. Note that the analytical result (35) for $b_{cr}(\delta_a/d)$ coincides with the numerically obtained plot (figure 4) with good accuracy (the relative deviation does not exceed 2%).

It is of special interest to study the case of strong non-linearity because in the limit $b \to \infty$ an explicit dependence of h on h_0 is obtained. Due to the fact that the dependence $h(h_0)$ has central symmetry, we can analyse only that part of the hysteresis loop where

$\operatorname{sign}\tilde{\kappa} = 1$ ($h > 0$). With $b \to \infty$ ($\alpha \to \infty$), the quantity μ (22) is equal to β/π and equation (29) for the induced field ($\kappa = h/\mathcal{H}$) gives

$$\kappa = \frac{(1-a^2)^{1/2}}{\beta} + 2(3)^{1/2}(\pi - \beta)(1-a^2)^{1/2}\left(\frac{\beta}{\pi}\right)^{4/3}\frac{\delta_a}{d}\sum_{n=1}^{\infty}\frac{1}{n^{1/3}(\beta^2 - n^2\pi^2)}$$
$$\cong \frac{(1-a^2)^{1/2}}{\beta} - 2(3)^{1/2}(1-a^2)^{1/2}\left(\frac{\beta}{\pi}\right)^{4/3}\frac{\zeta\pi^2 - (\zeta - 1)\beta^2}{\pi^2(\pi+\beta)}\frac{\delta_a}{d} \qquad (37)$$

where $\beta = \cos^{-1}(-a)$ and $\zeta = \zeta(\frac{7}{3}) \simeq 1.432$; $\zeta(x)$ is the Riemann zeta function.

We can see that in the case $\delta_a/d = 0$ the limit curve $\kappa(a)$ has universal character. Nevertheless, this result is not valid for a plate of finite thickness since the dependence $\kappa(a)$ (see equation (37)) contains an electrodynamic characteristic of the metal (δ_a) together with the plate thickness d.

4. Conclusion

The present investigation demonstrates that the magnetodynamic non-linearity mechanism leads to the effect of RF current rectification (effect of current state excitation) in metal plates of finite thickness. This effect is attributed to the periodical appearance and disappearance of a group of trapped electrons in the metal sample. The current state excitation manifests itself as a hysteresis dependence of the intrinsic magnetic moment of the plate on the external magnetic field. This phenomenon arises when the radio-wave amplitude exceeds a certain threshold value $\mathcal{H}_{cr}$. According to equations (2) and (35), the value $\mathcal{H}_{cr}$ depends on the parameters of the metal medium, radio wave frequency, temperature, and also on the plate thickness:

$$\mathcal{H}_{cr} \propto (cp_F\delta_a/el^2)(1 - 3^{1/2}\delta_a/d)^{-2}. \qquad (38)$$

With decreasing sample thickness d, the threshold of the current state generation shifts to large amplitudes of the incident wave. So, the smaller the sample thickness, the weaker is the current state effect.

It should be mentioned that the finiteness of the plate thickness modifies the frequency dependence of the threshold amplitude $\mathcal{H}_{cr}$. As follows from equation (38), this dependence will differ from the $\mathcal{H}_{cr} \propto \omega^{-1/3}$ law, which is valid when $d \to \infty$. Such deviations were observed in experiments [11, 13], carried out under conditions $\delta \simeq d$. Unfortunately, we cannot compare the theoretical and experimental results because of the lack of comprehensive information on the dependence $\mathcal{H}_{cr}(\omega)$, measured in [11] and [13]. To test our predictions it would be expedient to measure this dependence for a sample with diffuse surface in the case $\delta \simeq d$ once more and to compare the results with the formula (38).

Note, that the inequality (17) plays an important role in our theoretical consideration. In anomalously thin films or at very low frequencies, when the inequality (17) is not valid, the number of trapped electrons noticeably decreases. Simultaneously, the electromagnetic field 'x-rays' the sample, and the current state vanishes.

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